Abstract: The ability of rationalizing and representation is an ability needed by students in learning mathematics, to get student learning outcomes to be optimal. Therefore, the teacher can choose and apply an effective learning approach to improve students' rationalizing and representation abilities. This study aimed to determine the increase in students' rationalizing and ability through mathematics learning with the Open-Ended approach, to find out the differences of students’ rationalizing and representation ability between the students who got mathematics learning with the Open-Ended approach and ordinary mathematics learning, and the students’ responses who were taught by Open – Ended learning approach. The population of this study was the 10th grade students of AL Qomar Islamic Vocational School. The method used was an experimental method randomly selected two classes to be used as an experimental and control class. Based on the results of the study are concluded that the ability of rationalizing and representation ability of students who got mathematics learning with Open-Ended was better. And the data processing with level of 5% students’ rationalizing and representation of experimental class, who were taught using learning with the Open-Ended approach was higher than ordinary learning, with an average of experimental class 33.79 and control class 29.07. From the results of the questionnaire it can be concluded that the students' responses were very positive towards mathematics learning with the Open-Ended approach. Therefore mathematics learning with the Open-Ended approach can be used as an alternative to improve students' rationalizing and representation abilities.

Keywords: rationalizing, representation, open ended learning, and student attitudes.

Introduction

In the Principles and Standards for School Mathematics NCTM rationalizing and representation are two of the five abilities students should have. These abilities include problem solving, rationalizing, communication, connections and representation, in which these abilities have started from the kindergarten level. Inclusion of rationalizing and representation aspects in the standard learning process are two very important things because rationalizing is an abstract thinking activity.¹

Rationalizing and mathematics are two things which can not be separated, because mathematical material is comprehended through rationalizing and rationalizing is comprehended and trained through learning mathematics. This fact shows the mutual relationship between rationalizing with mathematics. Whereas representation is a mathematical language in the form of symbols, models, images or graphics. In accordance with the statement, mathematical
rationalizing is essential to increase the ability to present situations which are found through models, graphs or symbols.

Mathematical representation is one of the competencies in learning mathematics. Representation or model of a mathematical situation or concept, if it is presented in a ready-made form, can actually be seen as reducing or negating the students’ opportunity to think creatively and discover from the earlier mathematical concepts contained in a problem situation. Mathematics representation of the situation can appear in a variety of ways, concretes, mock objects or images, abstract art (sketches or symbols made by students themselves), as well as abstract symbols and formulas.²

Representation Standard in NCTM involves instructional programs as follows:³
a. Creating and using representations to organize, record, and communicate mathematical ideas;
b. Selecting, applying, and translating between mathematical representations to solve problems;
c. Using representations to model and interpret physical, social, and mathematical phenomena.

The students’ representation weaknesses are caused by how to present the symbols, images, models or graphics which are only considered as a complement in presenting the materials. This problem is found in every presentation of the mathematics learning material which makes students have lack of development and independent in finding knowledge.

Sabadar argues that Math concepts generally begin from the experience and events in human life, hence students can learn, understand, and comprehend the sense of situations or contexts which contain and create to a certain mathematics NCTM.

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4 The application of rationalizing is often found during the process of learning mathematics. Rationalizing and mathematics are two things which can not be separated,
because mathematics material is understood through rationalizing and rationalizing is understood and practiced through learning mathematics. This fact shows the mutual relationship between rationalizing with mathematics. This statement means that learning mathematics using rationalizing and practicing rationalizing using mathematics.

The phenomenon of vocational students have low ability in rationalizing and representation (as the results of a preliminary study class X students of Islamic Vocational School Al Qomar) that students who work on mathematical problems related to the ability of representation is weak, the rationalizing ability is also still low. This is caused by students who can not use the power of rationalizing ability to represent mathematics material which is abstract to concrete.

According to Piaget in Matlin, the age of vocational students for geometry is at the stage of concrete operations, making diagrams, images, and models. In this case, that the ability of students to manipulate concrete objects is supported by students' rationalizing power in representing mathematical ideas.

Mathematical learning can be improved by changing the ordinary learning method with one of methods that can improve the development of students' rationalizing and representation potential. The emergence of an open-ended approach begins with how to evaluate students' abilities in high-level thinking in mathematics objectively. An open-ended approach is a learning approach which presents a problem that has more than one answer or resolution method.

Nohda states that learning with an open-ended approach assumes three principles as follows: related to the autonomy of student activities; related to evolutionary anintegral nature of mathematical knowledge; and related to teachers' decision-making in class. In the open-ended approach, the teacher wisely gives students freedom to learn actively with minimal direction and the mathematics knowledge which is built naturally and thoroughly.

Implementation in the Open-ended approach, students are asked to solve problems accompanied by encouragement to develop their way of thinking and use problem-solving strategies which are convincing to them. This approach gives students freedom to investigate, construct and elaborate solutions to enable them increasing their problem solving abilities.

In the open-ended approach, the questions used should be able to be developed to complete mathematical knowledge. Sawada suggests a number of things that...
could be used as guidelines in the preparation of open-ended questions as follows:

a. Presenting problems through physical conditions, with the variables in the problem;
b. Presenting the forms (geometry) which students can make a conjecture;
c. Presenting a sequence of numbers or tables, which students can find the mathematical rules;
d. Give concrete examples in several categories which students can elaborate their properties to find general rules;
e. Give some similar exercises which students can generalize their work.

According to Sullivan, there are two methods to arrange problems in open-ended, namely: the method of working in reverse which includes three main steps, they are identifying topics, thinking questions and writing answers beforehand, and making open-ended questions based on these answers. The method of using standard question involves three main steps, such as identifying topics, thinking of standard questions, and creating open-ended questions based on predetermined standard questions.

In appropriate with its characteristic, questions in open-ended learning provide opportunities for students to answer the problem given with various answers. This can make difficulty for teachers to assess the results of student work. Sawada states that to overcome this, there are several criteria as follows:  

a. Proficiency, which is the ability to use several methods of resolution;
b. Flexibility, is an opportunity for students to answer correctly for several similar questions;
c. Authenticity, which measures the authenticity of students' ideas in giving the right answer.

In the learning process, students' attitudes also determine the success of learning. Norjoharuddeen argues that "Attitudes refers to the predisposition to respond in a favorable or unfavorable way with respect to a given object". It means, attitudes refer to one's tendency towards responses related to 'likes' or 'dislikes' to a given object. As the process of forming belief, the formation of a student's attitude towards mathematics requires a relatively long time.

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Beliefs and attitudes are formed little by little which is the result of student interaction with mathematics.\footnote{Norjoharuddeen}

Norjoharuddeen states that there are two factors which can influence the learning process of mathematics in every student, they are: (1) cognitive factors and (2) non-cognitive factors. Cognitive factors themselves are related to the brain's ability to think. Examples of cognitive factors are the ability to remember or reason. While non-cognitive factors are related to abilities outside the brain's ability to think.\footnote{Norjoharuddeen}

Students' attitudes towards mathematics can be in the form of positive attitudes which can help students to value mathematics and help students develop confidence in their abilities; while negative attitudes cannot help students to value mathematics and cannot help students develop confidence in their abilities.

This study generally aims to obtain information about the rationalizing abilities and mathematical representation of vocational students through Open-ended learning and ordinary learning. In more detailed objectives of this study are as follows:

1. Knowing the difference in rationalizing ability of groups of students who learn to use Open-ended learning approach with groups of students who learn to use ordinary learning approach.
2. Knowing the difference in the ability of representation groups of students who learn to use Open-ended learning approach with groups of students who learn to use ordinary learning approach.
3. Knowing learning with the Open-ended approach can improve students' mathematical rationalizing abilities than ordinary learning approach.
4. Knowing learning by using the Open-ended approach can improve students' representational abilities than the normal learning approach.
5. Knowing students' responses to the implementation of the Open-ended learning approach.

**Method**

In this research, the method used was the experimental method. This study was conducted to determine the effect of Open-ended learning treatment on increasing students' rationalizing and representation abilities.

The design of this study was the "non-equivalent control group" which is part of the "quasi-experimental" form. In this quasi experiment the subjects were not randomly grouped, but the researcher accepted the state of the subject in reality.\footnote{Sawada. Developing Lesson Plans. Dalam Jerry Becker dan Shigeru Shimada. The Open Ended} The use of the design in this study was done with the consideration that the existing class had been formed before, so that there were no more random groupings.

2 Ibid 12


The population in this study was the X grade students of Al Qomar Islamic Vocational School in the second semester of the 2018/2019 academic year consisting of one experimental class and two control classes that had taught using ordinary learning methods. The determination of the sample of this study had several criteria, such as: 1) The learning approach taken was generally ordinary learning, so that it lacked the creativity of students' thinking and students' independence to develop their rationalizing and mathematical representation abilities; 2) Class X was chosen with the assumption that they have just graduated from junior high school and have never experienced the Open-ended learning approach and do not interfere with the Open-ended learning approach to be studied; and 3) the presentation of mathematics lessons was at the end of learning so this condition was a challenge for researchers to motivate students to learn mathematics in pleasant situations by using the Open-ended learning approach.

The material chosen in this study was based on the subject of class X vocational high school which is the subject of linear program, which refers to the 2013 curriculum. Details of the Linear Program material are: 1) Determine the objective function; 2) Determine the optimum point of the set area of linear inequality system settlement; and 3) Determine the optimum value of the objective function.

The teaching materials were arranged in the form of worksheets which refered to Open-ended learning approach. In the process of teaching and learning activities, students are directed to find concepts in a variety of ways, as like by discussing through cooperative jigsaw groups, with peers and individually. Since the learning model was Open-ended, each meeting students had to experience the stages of Open-ended learning, such as: observation, questioning, guessing, collecting data and concluding.

The data collection in this study, using test and non-test techniques. The test technique was through tests of rationalizing and representation ability, while non-test techniques were attitude scale, teacher's questionnaire, observation sheet (student and teacher activities) and daily journal. Research questions were analyzed using descriptive statistics with percentage techniques; while the research hypotheses were answered using the n-test average difference (t-test) and assisted with SPSS 17.0 for Windows software program.

Results and Discussion

The data obtained and analyzed in this study were in the form of the results of the pretest, posttest and gain scores on aspects of
students' rationalizing abilities and mathematical representations.

The Pretest Results Mathematical rationalizing and representation ability of the Experiment Class and Control Class Students

Pretest Score of Students' Rationalizing and Mathematical Representation Abilities are scores obtained before given learning, both the experimental class and the control class.

From the results of the research showed the average results of the pretest in the experimental and control groups differed by 0.91 and the average ability of the experimental class was 3.84% of the ideal score. While the average control ability was 2.93% of the ideal score. Thus it could be stated that if it is analyzed from the average of the pretest percentage scores, the experimental group's rationalizing ability differs slightly.

Based on the results of data processing, it was obtained the lowest score, the highest score, and the average for the experimental and control groups, the ability of mathematical representation showed the average results of the pretest in the experimental and control groups differed by 0.82. If it is analyzed from the average percentage of pretest, the experimental class was 12.78% and the control class was 10.54%.

For the distribution of mathematical rationalizing abilities, the experimental class was more diffused than the experimental class because the standard deviation of the control class was greater, while the distribution of the mathematical representation ability of the experimental class was more diffused.

Based on the results of the normality test in the experimental group rationalizing ability obtained $x^2_{count} = 2,428$ dan $x^2_{table} = 11,341$, jadi $x^2_{count} < x^2_{table}$, then the pretest score of the experimental group was normally distributed, then $H_0$ was accepted. Normality test in the control group was obtained $x^2_{count} = 3,64$ dan $x^2_{table} = 11,341$ jadi $x^2_{count} < x^2_{table}$, then $H_0$ was accepted. This means that the control group pretest values were normally distributed.

The pretest data representation ability of the experimental group was obtained $x^2_{count} = 3,026$ dan $x^2_{table} = 11,341$ jadi $x^2_{count} < x^2_{table}$.

Based on the criteria of normality test, the pretest score of the experimental group's representation ability was normally distributed therefore $H_0$ was accepted. The normality test in the control group $x^2_{count} = 3,026$ dan $x^2_{table} = 11,341$ jadi $x^2_{count} < x^2_{table}$.

Based on the results of these calculations the pretest scores in the control group were normally distributed, therefore $H_0$ was accepted.

Based on the calculations, the scores of the experimental and control groups on the homogeneity test calculation of students'
rationalizing abilities were homogeneous or had the similar variant. The similar result was shown from the calculation of the homogeneity test score calculation of the ability of representation in the experimental and control classes that was homogeneous.

Pretest scores of both groups were normal and homogeneous. Furthermore, the similarity test of two averages was performed to find out the data values of the experimental and control groups were equal. The results obtained in the processing of data representation capability of t-count = 1,8002 and t-table = ± 2,389, thus H0 is accepted. Since −2.389 < 1,8002 < 2,389. This means that there is no significant difference in pretest scores between the experimental and control groups.

Posttest Results Mathematical rationalizing and representation ability of Students in Experiment Class and Control Class

The post-test for rationalizing ability and mathematical representation was done to find out the extent of students' rationalizing and mathematical representation ability after experiencing open-ended learning.

Based on the results of the large experimental class an average of 9,475 was greater than the control class of 5,6145. The mathematical representation ability of the experimental group 28.97% was greater than the average percentage of the posttest of the control group by 17.88%.

The processing data on the normality test in the experimental group showed that $X^2_{\text{count}} = 6,9601$ dan $X^2_{\text{table}} = 11,341$, sehingga $X^2_{\text{count}} < X^2_{\text{table}}$, thus the experimental class was normally distributed. Based on the calculation of the control group normality test obtained $X^2_{\text{count}} = 6,1616$ dan $X^2_{\text{table}} = 11,341$, sehingga $X^2_{\text{count}} < X^2_{\text{table}}$. This showed that the control group posttest score was normal then H0 was accepted.

Data processing in the experimental group for the ability of representation showed that $X^2_{\text{count}} = 3,0260$ dan $X^2_{\text{table}} = 11,341$, sehingga $X^2_{\text{count}} < X^2_{\text{table}}$. This means that the experimental class posttest scores for the representation ability was normally distributed. In the control group $X^2_{\text{count}} = 2,5726$ dan $X^2_{\text{table}} = 11,341$, sehingga $X^2_{\text{count}} < X^2_{\text{table}}$, this shows that the posttest score of control group was normally distributed, then H0 was accepted.

The results of homogeneity test calculations conducted in the experimental and control groups were found that $F_{\text{count}} = 1.6736$ and $F_{\text{table}} = 2.335$, this means that $F_{\text{count}} < F_{\text{table}}$, the experimental and control group on rationalizing ability were homogeneous or had the same variant. While the homogeneity test results on the representation ability performed was found that $F_{\text{count}} = 1.416$ and $F_{\text{table}} = 2.335$ then $F_{\text{count}} < F_{\text{table}}$. It can be concluded that the results of
the representation ability of the experimental group and the homogeneous of control group, then $H_0$ was accepted.

Based on testing hypotheses 1 and 2, it was found that the ability of rationalizing and representation in the experimental group was better than the control class. To find out the difference in increasing mathematical rationalizing ability groups of students who were taught using open-ended learning was better than a group of students who were taught using ordinary learning. The results obtained in the processing of $t_{count} = 4.3788$ and $t_{table} = 2.39$ then $H_0$ was rejected because $t_{count} > t_{table}$. It proves that there was a significant difference in the gaining score between the experimental group and the control group. In other words the improvement in rationalizing skills with open-ended learning is better than that which follows ordinary learning.

The calculation of the average difference test of students' mathematical representation ability obtained $t_{count} = 5.42718$ and $t_{table} = 2.39$ then $H_0$ was rejected. Because $t_{count} > t_{table}$ was not in the area of reception. This means that there was a significant difference in the gaining score between the experimental group and the control group. In other words the increased ability of the representation of groups of students who learn open-ended was better than ordinary learning.

**The Student Ability Illustrates Mathematical Ideas**

The ability of students to illustrate mathematical ideas, is observed from the results of the ability of rationalizing and representation. Based on the results of the study it appeared that the pre-test age of the experimental group was better than the control group. This means that the experimental group with open-ended learning was better at illustrating mathematical ideas than the control group and the students' representation ability was 3.125% better than rationalizing ability.
The Students’ Ability to Solve Mathematical Problems Through Models

The ability of students to write mathematical models, observed from the results of the ability to reason and representation. Questions which support how the students' ability to illustrate mathematical ideas were seen in the post-test questions of rationalizing ability 2 and number 3, while those in representation ability were seen in questions number 3 and normor 4. Students' rationalizing ability in illustrating mathematical ideas was superior to the matter of representation. Based on the results of the study the students’ representation ability to solve problem number 4 was very low at only 18.8%.

The Attitude Scale Results

The results of the student attitude scale showed that the students' attitude to be pleased with mathematics, 14 people who strongly agreed and 14 people who agreed with number 1. This means that 28 people responded to the positive statement from 30 participants.

The favorite indicator of mathematics with the statement "I like learning mathematics" presented a positive response of 46.7% strongly agree and 46.7% for the choice of agree. If it is viewed from the student attitude score, the student attitude score was greater when compared to the national score of 4.27> 2.25.

From the findings above, it can be concluded that students' attitudes towards mathematics in terms of their interest for participation in discussions and the usefulness of mathematics in daily life generally showed positive attitudes. Students responded positively to these statements also influenced by teaching methods which are tailored to the topic of the material being taught and the teacher's attitude in implementing the Linear Program.

The findings in this study that in general the attitude of the students most of the students really liked the activities contained in open-ended learning. While the analysis of the scale of attitudes towards representation and rationalizing can be concluded that the overall attitude of students towards learning mathematics was positive. This can be seen from the student attitude score 4.4% greater than the neutral score of 2.75. Students attitudes toward learning open-ended scores produced by 4 times greater than the neutral score. Students' attitudes toward rationalizing problems seen in the acquisition of a scale score of 4 was greater than the student's neutral attitude score of 2.94. Student attitude scores on representation questions by 4 was greater than the average student attitude score of 3.

Observation Results

Based on the results of the study, the average activity of the most dominant
students was the seriousness of doing homework that was (100%). Furthermore, the more dominant activity was paying attention to the teacher's explanation by 90%, discussing between groups that was 87.5%. Overall student activity in open-ended learning was in the good category.

The results showed that the experimental class that experienced treatment with open-ended learning activities was better than the control class, because the teacher did not instruct the students in discussing, expressing opinions, and solving questions of rationalizing and representation. This condition allowed the score achieved by the control class was lower than the experimental class to solve the problem of rationalizing and representation.

Fifth, the open-ended learning model can significantly improve the ability of rationalizing and representation.

This research recommendation is addressed to parties related to the results of the study, as follows:
1. Vocational School Math Teachers in class X can develop, update, or follow up on learning models to improve rationalizing and representation skills in accordance with the applicable curriculum.
2. Researchers Next, can review and re-test the effectiveness of learning models to improve students' more varied rationalizing and representation abilities.

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